Parallelizing the Mur$\varphi$ Verifier

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Abstract. With the use of state and memory reduction techniques in verification by explicit state enumeration, runtime becomes a major limiting factor. We describe a parallel version of the explicit state enumeration verifier Mur$\varphi$ for distributed memory multiprocessors and networks of workstations that is based on the message passing paradigm. In experiments with three complex cache coherence protocols, parallel Mur$\varphi$ shows close to linear speedups, which are largely insensitive to communication latency and bandwidth. There is some slowdown with increasing communication overhead, for which a simple yet relatively accurate approximation formula is given. Techniques to reduce overhead and required bandwidth and to allow heterogeneity and dynamically changing load in the parallel machine are discussed, which we expect will allow good speedups when using conventional networks of workstations.

1 Introduction

Complex protocols are often verified by examining all reachable protocol states from a set of possible start states. This reachability analysis can be done using two different methods: the states can be explicitly enumerated by storing them individually in a table, or a symbolic method can be used, such as representing the reachable state space with a binary decision diagram (BDD) [3]. Both methods have application domains in which they outperform the other; explicit state enumeration has worked better for the types of industrial protocols examined in our group [11].

There have been two approaches to improve explicit state enumeration. First, state reduction methods have been developed that aim at reducing the size of the reachability graph while ensuring that protocol errors will still be detected. Examples would be exploiting symmetries, utilizing reversible rules and employing repetition constructors [13]. These methods directly tackle the main problem in reachability analysis: the very large number of reachable states of most protocols. The second approach aims at reducing the amount of memory needed to perform the reachability analysis. Examples would be bitstate hashing [9] and hash compaction [26, 20].

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In this paper, we explore a third approach to improve explicit state enumeration: parallel processing. With the use of state and memory reduction techniques, runtime becomes a major limiting factor [26, 20]. For example, when verifying complex protocols with the Murϕ verifier [7] using symmetry reduction in combination with hash compaction, a single verification run that does not expose new errors typically takes several days.

We present a parallel version of the Murϕ verifier for distributed memory multiprocessors and networks of workstations that uses the message passing paradigm. Parallel Murϕ was originally developed on a network of workstations (NOW) at UC Berkeley (SPARC-20s connected via Myrinet) using generic active messages [6] as the message passing layer; later it was ported with little effort to an SP2 at IBM Watson.

In parallel Murϕ, the state table, which stores all reached protocol states, is partitioned over the nodes of the parallel machine. Thus, the table can be larger than on a single node. Each node maintains a work queue of unexplored states. When a node generates a new state, the “owning” node for this state is calculated with a hash function and the state is sent to this node; this policy implements randomized load balancing. On reception of a state descriptor, a node first checks if the state has been reached before (with the local part of the state table). If the state is new, it is inserted in the state table and the local work queue. Special algorithms for termination detection and error trace generation have to be employed in this distributed setting. Due to space constraints, however, only the termination detection algorithm will be discussed in this paper. We also show analytically that the state space is typically very evenly distributed over the nodes.

We measured the speedup of parallel Murϕ when verifying three complex cache coherence protocols: SCI [12], DASH [17] and FLASH [16]. On a 63 node SP2 the speedup was 44.2 for SCI and 53.7 for DASH, while we obtained speedups of 26.6 for SCI, 27.8 for DASH, and 29.4 for FLASH on a 32 node NOW in Berkeley. Thus, our algorithm achieves close to linear speedup. In addition, experiments performed at Berkeley [18] show that the runtime of parallel Murϕ is largely insensitive to increased communication latency and reduced bandwidth. There is, however, some sensitivity to communication overhead. We give a simple formula for the expected runtime on a parallel machine depending on the communication overhead. We show empirically that the formula accurately predicts parallel speedup.

Aggarwal, Alonso and Courcoubetis [1] also presented a distributed reachability algorithm. Their algorithm seems more complicated than ours, has not been implemented and the correctness of the termination detection relies on timing assumptions that may be difficult to guarantee. One potential advantage of their method is that it might be usable under dynamically changing load conditions on a network of heterogeneous workstations. We propose an extension of our algorithm, however, that also allows heterogeneous systems with dynamically changing load and, at the same time, reduces the communication volume by typically one or two orders of magnitude.
Kumar and Vemuri [15] proposed and implemented a distributed algorithm to check the equivalence of two finite state machines, which essentially does a reachability analysis of the product machine. Their algorithm synchronizes after each breadth-first level and does not overlap communication and computation. Although the examples they present require only infrequent communication with very small messages, the reported speedups are worse than the ones reported here. In addition, their algorithm seems to have a high overhead, since it is only faster than a sequential one when running on four nodes.

Parallel Murϕ, however, when running on one node is as fast as the most recent version (3.0) of sequential Murϕ, for which the runtime was optimized. In fact, parallel Murϕ is based on this version of sequential Murϕ, which contains symmetry reduction and hash compaction.

There have also been some efforts to parallelize BDD-based verification methods. Stornetta and Brewer [23, 22] and Ranjan et al. [19] have presented distributed memory BDD algorithms. Both algorithms only achieve speedups in comparison to sequential versions if the sequential versions run out of memory and are forced to do swapping, but they enable the use of the total memory of the parallel machine. Kimura and Clarke [14] presented BDD algorithms for a shared memory machine and reported a speedup of roughly 10 on 15 nodes, while efficiently using the total available memory.

2 Active Messages

Active messages [24] have been introduced to reduce the communication costs in message passing and can be thought of as a fast message passing library. In contrast to a message in traditional message passing, an active message also contains the address of a procedure, called handler, that will be called on the destination node after the arrival of the message. For example, when a state descriptor $s$ is to be sent to some node $n$, we will send the active message “Receive($s$)” to $n$, indicating that the handler Receive() should be called on node $n$ with the state descriptor $s$ as argument.

When sending an active message, the sender does not wait for the message to arrive at the receiver but continues immediately. Upon arrival of the message, the receiver’s current stream of control is not interrupted. Instead, the receiver has to periodically call poll(), which, in turn, calls all handlers for the active messages that have arrived since the last call to poll(). We will say that a message is received after the corresponding handler has returned. The use of handlers and polling enables efficient implementations of the active message scheme.

All nodes have to execute the same program when using active messages. Each node, however, is assigned a unique node number from $\{0, \ldots, N-1\}$, where $N$ denotes the number of nodes in the parallel machine. To implement a “master” node with special responsibilities like, for example, startstate generation, an if statement can be used with the condition that the node number is 0. The barrier() command synchronizes all the nodes running the parallel program by waiting until every node has reached the barrier.
3 Parallel Explicit State Enumeration

The Basic Algorithm

The basic algorithm that runs on each node of the parallel machine is given in Figure 1 and described in this paragraph. Note that the global variables are local to each node since we assume distributed memory. The state enumeration is started by calling Search() on each node. The master node generates the start states and distributes them by calling Send(). In the Send() procedure, the state is first canonicalized (for symmetry reduction) and then sent to the owning node, whose node number is calculated by a hash function $h()$. The handler Receive() checks a state against the local state table and potentially inserts it into the state table and queue. The search loop dequeues a state, generates its successors and sends them to the owning nodes. Note that this loop calls poll() to execute the handlers for newly arrived messages. The search loop is exited as soon as termination is detected. Termination detection is described in more detail in the next subsection.

Termination Detection

The parallel search has terminated when the following two conditions hold: there are no more messages in progress (i.e. sent but not received) and there are no more states in the queues $Q$. Note that the latter condition also implies that no state is currently being expanded, since states being expanded are removed from the queue only after their expansion.

Figure 2 shows the termination detection algorithm used. The algorithm is only invoked after the master has been idle for longer than a certain threshold value. Setting this value to, say, 0.1s results in negligible runtime overhead for termination detection. The correctness proof is similar to the one presented in [25] and is omitted here.

Randomized Load Balancing

We now examine how well the hash function balances the state space over the individual state tables. We look at a particular node and assume that for each state the probability that it is sent to this node is $1/N$, i.e., that the hash function distributes states uniformly. (Universal hashing [4], used in Murcy [20], can be shown to distribute at least as well as uniformly.) Let $n$ denote the number of reachable states and $Y$ the random variable describing the number of states sent to our node, which has the expected value $\hat{Y} = n/N$. For a large $n$, $Y$ is distributed according to a normal distribution because of the central limit theorem. To bound the probability that the relative error of $Y$ in comparison to $\hat{Y}$ is larger than a certain constant $r$, we use that for every $x > 0$, $1 - \Phi(x) < \phi(x)/x$, where $\Phi(x)$ and $\phi(x) = e^{-x^2/2}/\sqrt{2\pi}$ denote the standard normal distribution and density functions [8]. Using basic calculations one obtains that

$$\Pr(|Y/\hat{Y} - 1| > r) < 2 \phi(z)/z \quad \text{with} \quad z = r \sqrt{n/(N-1)}.$$
var // global variables, but local to each node
T: hash table; // state table
Q: FIFO queue; // state queue
StopSend: boolean; // for termination detection
Work, Sent, Received: integer;

Search() // main routine
begin
T := ∅; Q := ∅; // initialization
StopSend := false; Sent := 0; Received := 0;
barrier();
if I am the master then // master generates startstates
  for each startstate s₀ do
    Send(s₀);
  end
do // search loop
  if Q ̸= ∅ then begin
    s := top(Q);
    for all s' ∈ successors(s) do
      Send(s');
  end
  Q := Q - {s};
  poll();
  while not Terminated();
end

Send(s: state) // send state s to “random” node h(s)
begin
  s_c := canonicalize(s); // symmetry reduction
  while StopSend do // wait for StopSend = false
    poll(); // (for termination detection)
  end
  Sent++;
  send active message Receive(s_c) to node h(s_c);
end

Receive(s: state) // receive state (active message handler)
begin
  Received++;
  if s ∉ T then begin
    insert s in T;
    insert s in Q;
  end
end

Fig. 1. Parallel Explicit State Enumeration
Terminated(): boolean
begin
if I am the master then    // master initiates termination check
  if idletime exceeds threshold then begin
    Work := 0;
    send active message ReportCounters() to all nodes;
    wait for all replies (i.e., calls to SumCounters());
    if Work > 0 then        // continue search
      send active message Continue() to all nodes;
    else                    // terminate search
      notify all nodes of termination (details omitted);
  end
return termination status;
end

// active message handlers
ReportCounters()       // report counter values
begin
  StopSend := true;
  send active message SumCounters(Sent - Received + |Q|) to master;
end
SumCounters(w: integer)      // master sums counter values
begin
  Work := Work + w;
end
Continue()                // continue with search
begin
  StopSend := false;
end

Fig. 2. Termination Detection

For example, when \( n=10^8 \) and \( N=32 \), the probability that the relative error exceeds \( r=0.1\% \) is smaller than 8.85% and the probability that the relative error exceeds \( r=0.5\% \) is smaller than \( 2.73 \times 10^{-19} \). Generally, if the number of reachable states \( n \) is large and the number of nodes \( N \) is not too large, the state space will be distributed very evenly over the nodes.

Results

Figures 3 and 4 show the measured speedup of parallel Murϕ on a 63-node SP2 at IBM Watson and on a 32-node UltraSPARC/Myrinet NOW at Berkeley, for instances of the SCI, DASH and FLASH protocols. Some parameters of these instances are shown in Table 1. The protocols were scaled to both provide interesting data and make the process of running the examples not too time-consuming. The speedup graphs show that the Murϕ verifier can be parallelized quite efficiently.
Table 1. Example protocols

<table>
<thead>
<tr>
<th>protocol</th>
<th>reachable states</th>
<th>successors generated</th>
<th>bytes/state</th>
<th>diameter</th>
<th>single-node runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCI</td>
<td>1179942</td>
<td>2973536</td>
<td>124</td>
<td>46</td>
<td>717s</td>
</tr>
<tr>
<td>DASH</td>
<td>254937</td>
<td>2646647</td>
<td>532</td>
<td>64</td>
<td>1287s</td>
</tr>
<tr>
<td>FLASH</td>
<td>1021464</td>
<td>4556496</td>
<td>136</td>
<td>45</td>
<td>2477s</td>
</tr>
</tbody>
</table>

Fig. 3. Speedups for the SCI (dotted) and DASH (solid) protocols, calculated from the average runtime over two runs on an SP2, in comparison to linear speedup (dashed)

Fig. 4. Speedups for the SCI (dotted), DASH (solid) and FLASH (dashed and dotted) protocols, calculated from the average runtime over five runs on the Berkeley NOW, in comparison to linear speedup (dashed)
4 Estimating the Speedup

Rich Martin et al. [18] have performed a study on the Berkeley NOW of the impact of communication performance on several parallel applications including Murϕ. They characterized communication performance based on the LogGP model [5, 2] with four parameters: latency $L$, overhead $o$, gap $g$ and time-per-byte $G$. The latency is the delay in communicating a small message, the overhead is the average time consumed in sending or receiving a message, the gap is the minimum time between two messages, i.e. the reciprocal message bandwidth, and the time-per-byte is the reciprocal bulk transfer bandwidth.

The communication layer of the NOW was modified so that each of these parameters could be slowed down independently, starting from the following values of the unmodified communication layer: $o=3.5\mu s$, $g=7.0\mu s$, $L=5.5\mu s$ and $G=38MB/s$. Parallel Murϕ, when verifying the SCI example, showed only negligible slowdown when either increasing latency or gap by up to $100\mu s$ or when reducing the bulk transfer bandwidth to $1MB/s$. The insensitivity to latency can be explained by observing that if there are enough states in the state queues, all latency is overlapped with computation. Parallel Murϕ is not sensitive to increased gap since it does not send messages in bursts. Finally, the bandwidth requirement per node is smaller than $1MB/s$ (roughly $0.5MB/s$).

The runtime of parallel Murϕ, however, showed some dependency on the overhead, for which we now derive an approximation formula. We assume that each node sends $m/N$ messages, where $m$ denotes the total number of messages sent. On the average, a fraction of $(N - 1)/N$ of these messages will be sent to nodes different from the sender, each resulting in an overhead of $4o$, which stems from the sending and receiving of the message and its (automatically generated) reply message. Assuming linear speedup if there were no overhead, we approximate the runtime $t_N$ on $N$ nodes as

$$t_N = 4o m (N - 1)/N^2 + t_1/N,$$

where $t_1$ denotes the runtime on a single node. Table 2 shows that (1) quite accurately predicts the measured runtimes for a range of different overhead values and numbers of nodes. Note that the numbers of messages sent are (slightly) smaller than the number of generated successors ($2.974 \cdot 10^6$), which is due to a small cache of recently sent states.

5 Improvements of the Basic Algorithm

Message Aggregation

By packing several states into one message, one can reduce the overhead per state. This well-known technique basically trades excess parallelism for communication performance. As shown in Figure 5, each of our three sample protocols provides a high degree of parallelism measured in the number of states in each
Table 2. Measured \( (t_{N,m}) \) and predicted \( (t_{N,p}) \) runtimes for the SCI protocol (in seconds) when varying the overhead. Measurements are averaged over five runs.

<table>
<thead>
<tr>
<th>( N )</th>
<th>messages sent [million]</th>
<th>( t_{N,m} )</th>
<th>( t_{N,p} )</th>
<th>( t_{N,m} )</th>
<th>( t_{N,p} )</th>
<th>( t_{N,m} )</th>
<th>( t_{N,p} )</th>
<th>( t_{N,m} )</th>
<th>( t_{N,p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.983</td>
<td>705.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.534</td>
<td>372.2</td>
<td>361.8</td>
<td>432.1</td>
<td>425.1</td>
<td>496.1</td>
<td>488.5</td>
<td>617.4</td>
<td>615.2</td>
</tr>
<tr>
<td>16</td>
<td>2.697</td>
<td>52.7</td>
<td>46.3</td>
<td>66.5</td>
<td>62.1</td>
<td>81.6</td>
<td>77.9</td>
<td>114.1</td>
<td>109.5</td>
</tr>
<tr>
<td>32</td>
<td>2.706</td>
<td>26.2</td>
<td>23.2</td>
<td>34.6</td>
<td>31.4</td>
<td>43.1</td>
<td>39.6</td>
<td>59.4</td>
<td>56.0</td>
</tr>
</tbody>
</table>

Fig. 5. Number of states in each breadth-first level for the SCI (dotted), DASH (solid) and FLASH (dashed and dotted) protocols.

level of a breadth-first search, which enabled the efficient parallelization in the first place.

Table 3 shows the effect of message aggregation on runtime \( t_N \) and the number of messages sent \( m \) for the unmodified NOW and with an additional overhead of 100\( \mu s \). An overhead of 100\( \mu s \) is typical for message passing libraries based on TCP/IP. In both cases, the number of messages sent is strongly reduced. Our implementation packs 10 states into each message given that there are more than 20 states in the local queue. We have not tried to optimize these values or even make the message size vary with the number of states in the queue. While the reduction in runtime is small in the case of the unmodified NOW (as expected), in the high overhead case we achieve approximately a factor of two reduction, approaching the runtime on the unmodified NOW.

Aiming for a Conventional NOW

The bandwidth requirement of parallel Murph becomes a problem on a conventional NOW, like workstations connected by Ethernet. For example, when veri-
Table 3. Measured \( (t_{N,m}) \) and predicted \( (t_{N,p}) \) runtimes and messages sent \( m \) (in million) for the SCI protocol when using message aggregation in comparison to the basic scheme. Measurements are averaged over six runs.

<table>
<thead>
<tr>
<th>( N )</th>
<th>( t_{N,m} )</th>
<th>( m )</th>
<th>( t_{N,m} )</th>
<th>( m )</th>
<th>( t_{N,m} )</th>
<th>( m )</th>
<th>( t_{N,m} )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>52.3s</td>
<td>2.692</td>
<td>47.9s</td>
<td>.334</td>
<td>43.9s</td>
<td></td>
<td>114.1s</td>
<td>2.692</td>
</tr>
<tr>
<td>32</td>
<td>25.7s</td>
<td>2.708</td>
<td>25.9s</td>
<td>.400</td>
<td>22.0s</td>
<td></td>
<td>58.9s</td>
<td>2.703</td>
</tr>
</tbody>
</table>

Focusing the DASH protocol, each node requires a bandwidth of roughly 0.5MB/s, which would make an implementation even on top of switched Ethernet (where each node gets 10MB/s for itself) difficult. In addition, the presented algorithm only works optimally if the state table size is proportional to the speed for each of the nodes, and thus allows only limited heterogeneity. Also, randomized load balancing performs poorly if the load on the nodes changes dynamically, since the hash function is fixed.

The algorithm can, however, be adapted to the situation of a slow network, heterogeneity and dynamically changing load. Instead of sending a full state \( s \), each node only sends a (hash) signature \( c(s) \) to the owning node \( h(s) \), which, in turn, returns a bit indicating whether the state had been reached before. It can be shown in a similar fashion to [20] that this scheme will typically enable a reduction in bandwidth requirements of one or two orders of magnitude at the cost of a small probability that the search becomes incomplete. Note that in this scheme each node generates its work (new states to explore) by itself, which has the effect that a fast node needing much work will also generate much work for itself. Thus, the scheme is well suited for situations of dynamically changing load. To provide each node with initial work and to “restart” a node that runs out of states, a load balancing protocol similar to the one described in [25] can be employed. Also, heterogeneous systems are allowed since the tabulation of states is now independent of their expansion.

6 Conclusions and Future Research

Runtime has been becoming a major bottleneck in verification. We show that the Mur\( \varphi \) verifier can be parallelized quite efficiently. The resulting algorithm is shown to run with close to linear speedup on a wide range of distributed memory multiprocessors and networks of workstations. In addition, we give a formula with which the speedup of parallel Mur\( \varphi \) can be predicted depending on the communication performance. Since the state table is partitioned over the parallel machine, the algorithm also allows the verification of larger protocols.

The methods used to parallelize Mur\( \varphi \) could also be used for other explicit state verification tools like SPIN [10]. The architectures for which parallel Mur\( \varphi \)
was developed – distributed memory multiprocessors and networks of workstations – are becoming more common. Techniques to reduce overhead and required bandwidth and to allow heterogeneity and dynamically changing load in the parallel machine are discussed, which we expect will allow good speedups when using conventional networks of workstations.

The algorithm presented is compatible with the two newer reduction techniques in Murϕ [13], reversible rules and repetition constructors, which were not yet available in the public release. It is also compatible with the latest version of hash compaction [21].

The most recent version of sequential Murϕ, on which the parallel version is based, does not support the checking of temporal properties because of difficulties combining this checking with symmetry reduction. Thus, we did not put high priority on parallelizing the verification of temporal properties. This seems, however, to be an interesting area for future research.

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